$$r = \frac{l}{1 + e\cos\theta}$$

$$\therefore (1 + e\cos\theta) = l$$

$$r = l - er\cos\theta = l - ex$$

$$r^2 = l^2 - 2lex + e^2x^2$$

$$x^2 + y^2 = l^2 - 2lex + e^2x^2$$

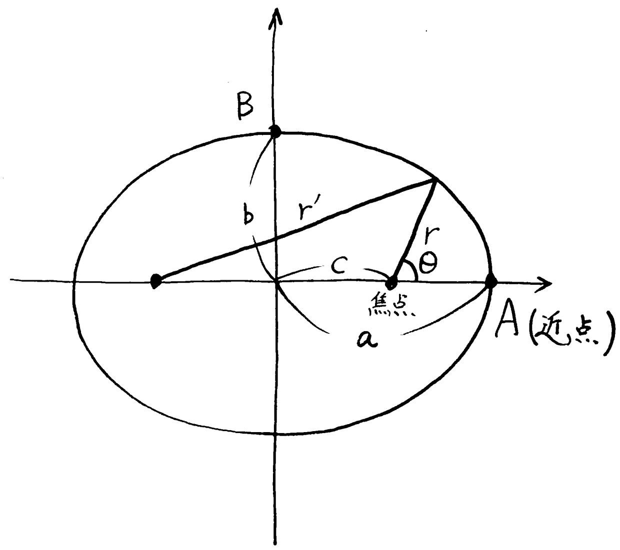
$$(1-e^2)x^2 + 2lex + y^2 = l^2$$

$2lex + y^2 = l^2$ ：放物線 —— ①

- $e < 1$ のとき $(1 - e^2 > 0)$

$$x^2 + \frac{2le}{1-e^2}x + \frac{y^2}{1-e^2} = \frac{l^2}{1-e^2}$$

$\left(x + \dfrac{le}{1-e^2}\right)^2 + \dfrac{y^2}{1-e^2} = \dfrac{l^2}{1-e^2} + \dfrac{l^2e^2}{(1-e^2)^2}$ ：楕円 —— ②



$r + r' = $ 一定

a：長半径

c：焦点距離

b：短半径

A点では

$$r + r' = (a-c) + (a+c) = 2a$$

B点では、

$$r + r' = 2\sqrt{b^2 + c^2}$$

$$\therefore a = \sqrt{b^2 + c^2}$$

$$\therefore c = \sqrt{a^2 - b^2}$$

離心率

$$e = \frac{c}{a} = \sqrt{1 - \frac{b^2}{a^2}}$$

②に代入すると、

$$\left(x + \frac{\frac{lc}{a}}{1 - \frac{c^2}{a^2}}\right)^2 + \frac{y^2}{1 - \frac{c^2}{a^2}} = \frac{l^2}{1 - \frac{c^2}{a^2}} + \frac{\frac{l^2 c^2}{a^2}}{\left(1 - \frac{c^2}{a^2}\right)^2}$$

$$\therefore \frac{\left(x + \frac{lac}{b^2}\right)^2}{a^2} + \frac{y^2}{b^2} = \frac{l^2}{b^2} + \frac{b^2 c^2}{b^4}$$

$\ell = \dfrac{b^2}{a}$ とすると、

$$\frac{(x+c)^2}{a^2} + \frac{y^2}{b^2} = 1$$

◎ $e > 1$ のとき $(e^2-1>0)$

$$(e^2-1)x^2 - 2\ell e x - y^2 = -\ell^2$$

$$\left(x - \frac{\ell e}{e^2-1}\right)^2 - \frac{y^2}{e^2-1} = -\frac{\ell^2}{e^2-1} + \frac{\ell^2 e^2}{(e^2-1)^2}$$ ：双曲線 ― ③

$$e = \frac{c}{a},\quad c^2 = a^2 + b^2$$

$$\ell = \frac{b^2c^2}{a^2} - b^2 = \frac{b^4}{a^2}$$

これを③に代入

$$\left(x - \frac{\frac{b^2}{a}\cdot\frac{c}{a}}{\frac{b^2}{a^2}}\right)^2 - \frac{y^2}{\frac{b^2}{a^2}} = -\frac{b^4a^2}{b^2a^2} + \frac{\frac{b^4}{a^2}\cdot\frac{c^2}{a^2}}{\frac{b^4}{a^4}}$$

$$\therefore \frac{(x-c)^2}{a^2} - \frac{y^2}{b^2} = 1$$ 双曲線

○ 問題1.23

8/6 (火) 75分