$$A = \begin{pmatrix} \boxed{v_1} & \cdots & \boxed{v_i} & \cdots \end{pmatrix}$$ n次正方行列

$$\det A = v_1 * v_2 * \cdots * v_n$$

$$v_1 * \cdots * v_{i-1} * (\lambda u + w) * v_{i+1} * \cdots * v_n$$

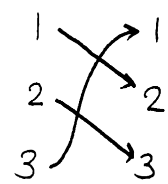
$$= \quad v_1 * \cdots * v_{i-1} * (\lambda u) * v_{i+1} * \cdots * v_n$$

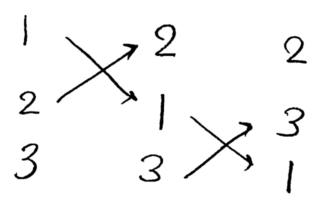
$$+ \; v_1 * \cdots * v_{i-1} * (w) * v_{i+1} * \cdots * v_n$$

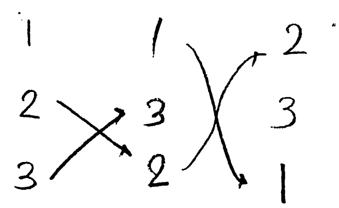
$$\det \begin{pmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ x_1^2 & x_2^2 & x_3^3 \end{pmatrix} = \sum_{\sigma \in S_n} \varepsilon(\sigma) \left(\sum_{k=1}^{3} a_{k\sigma(k)} \right)$$

$(1)\quad (1\ 2)\quad (2\ 3)\quad (3\ 1)$ 同じ$\left\{\begin{matrix}(1\ 2\ 3)\\ 3\ 1\ 2\\ 2\ 3\ 1\end{matrix}\right.$ 同じ$\left\{\begin{matrix}(2\ 1\ 3)\\ 3\ 2\ 1\\ 1\ 3\ 2.\end{matrix}\right.$ の 6置換

$$\begin{aligned}1 &\to 2\\ 2 &\to 3\\ 3 &\to 1\end{aligned}$$







$$\left.\begin{aligned}(1\ 2\ 3) &= (1\ 3)(1\ 2)\\ &= (1\ 2)(2\ 3)\end{aligned}\right\}\ \text{一意ではない.}$$

しかし、互換の$\left(\begin{matrix}\text{個数?}\\ \text{個数の偶奇?}\end{matrix}\right)$は一意、

であることを示せ。

同じことをくりかえすことを含めると、偶奇が正確か。

$$\det\begin{pmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ x_1^2 & x_2^2 & x_3^2 \end{pmatrix} = 1\cdot x_2\cdot x_3^2 - 1\cdot x_1\cdot x_3^2 - 1\cdot x_3\cdot x_2^2$$

$$- 1\cdot x_2\cdot x_1^2 + 1\cdot x_3\cdot x_1^2 + 1\cdot x_1\cdot x_2^2$$

$$= x_1\left(-x_3^2 - x_2x_1 + x_3x_1 + x_2^2\right)$$

$$+ x_2x_3\left(x_3 - x_2\right)$$

$$= x_1\left\{(x_2+x_3)(x_2-x_3) + x_1(x_3-x_2)\right\}$$

$$+ x_2x_3\left(x_3 - x_2\right)$$

$$= x_1\left(x_3-x_2\right)\left(x_1 - x_2 - x_3\right) + x_2x_3\left(x_3 - x_2\right)$$

$$= \left(x_3-x_2\right)\left\{x_1\left(x_1 - x_2 - x_3\right) + x_2x_3\right\}$$

$$= \left(x_3-x_2\right)\left(x_2(x_3 - x_1) + x_1(x_1 - x_3)\right)$$

$$= \left(x_3-x_2\right)\left(x_3 - x_1\right)\left(x_2 - x_1\right)$$

$$= -\left(x_1-x_2\right)\left(x_2-x_3\right)\left(x_3 - x_1\right)$$

$$\binom{n}{m} = \frac{n \times \cdots \times (n-m+1)}{m!} = \frac{n!}{m!(n-m)!}$$

$$= \frac{n \times \cdots \times \{n-(m-1)\}}{m!}$$

$$\binom{-1}{m} = \frac{(-1)\times(-2)\times\cdots\times\{\overbrace{(-1)-(m-1)}^{(-m)}\}}{m!}$$

$$= \frac{(-1)^m \cdot m!}{m!} = (-1)^m$$

$$\frac{1}{1+X} = \sum_{n=0}^{\infty} (-1)^n X^n = 1 - X + X^2 - X^3 + X^4 - \cdots$$

$$(1+X)^{-1} = \sum_{n=0}^{\infty} \underbrace{\binom{-1}{n}}_{(-1)^n} X^n$$

$$a_{ij} = \binom{x+i-1}{j-1}$$

$$a_{11} = \binom{x}{0} = 1 \ ?$$

$$a_{21} = \binom{x+1}{0} = 1 \ ?$$

$$a_{12} \ \binom{x}{1} = \frac{x}{1!}$$

$$a_{22} = \binom{x+1}{1} = \frac{x+1}{1!}$$

$$a_{13} = \binom{x}{2} = \frac{x(x-1)}{2!}$$

$$a_{23} = \binom{x+1}{2} = \frac{(x+1)x}{2!}$$

$$n\left\{\begin{pmatrix} 1 & x & \frac{x(x-1)}{2!} & \frac{x(x-1)(x-2)}{3!} & \cdots \\ 1 & x+1 & \frac{(x+1)x}{2!} & \frac{(x+1)x(x-1)}{3!} & \\ 1 & x+2 & \frac{(x+2)(x+1)}{2!} & \frac{(x+2)(x+1)x}{3!} & \\ 1 & x+3 & \frac{(x+3)(x+2)}{2!} & \frac{(x+3)(x+2)(x+1)}{3!} & \\ \vdots & & & & \end{pmatrix}\right.$$

項の数は $n!$

分母はみな $\prod_{k=0}^{n-1} k!$

分子の次数は、$\sum_{k=0}^{n-1} k$

$$\left(\begin{array}{ccc|ccc}
a_{11} & \cdots & a_{1,j-1} & a_{1,j+1} & \cdots & a_{1n} \\
\vdots & & & & & \\
a_{i-1,1} & & & & & \\
\hline
a_{i+1,1} & & & & & \\
\vdots & & & & & \\
a_{n1} & & & & &
\end{array}\right) = A_{ij}$$